Note Title 9/14/2006

Lecture 4

Given a set S

Manifold or a surface:

a map (diffeomorphism) (smooth) $\mathbb{R}^n \xrightarrow{\phi} \mathbb{R}_m$ C^∞

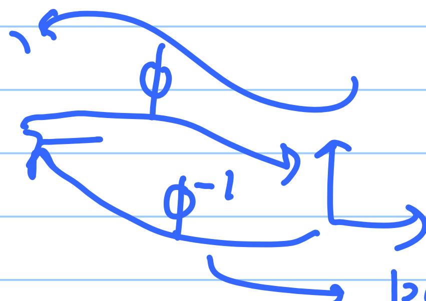
X open set $\subset \mathbb{R}^n$ $V \xrightarrow{\psi} Y$ X open

$U \subseteq S$ open

U

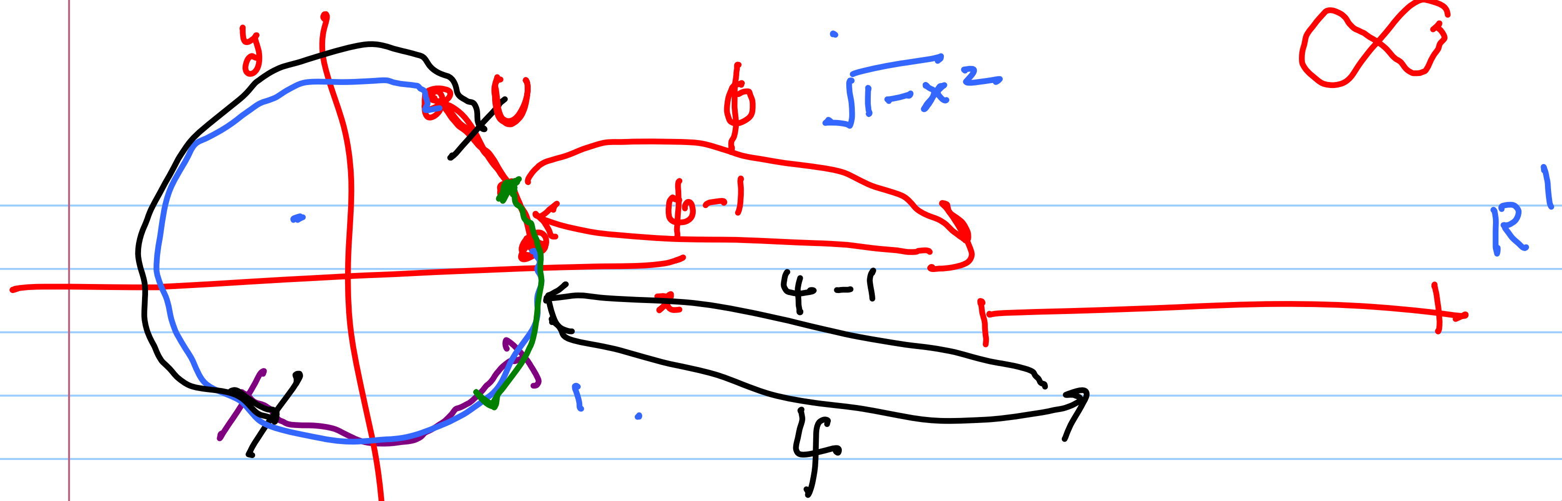
$\underline{\underline{U}} \xrightarrow{\phi} X \subseteq \mathbb{R}^m$

$= \Theta \leftarrow$ unit sphere S^2



$\phi^{-1} \longrightarrow$ parametrization

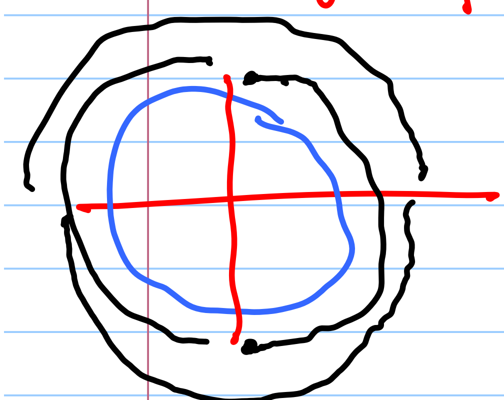
Chart: ϕ, U



$x^2 + y^2 = 1$

U, ϕ : chart

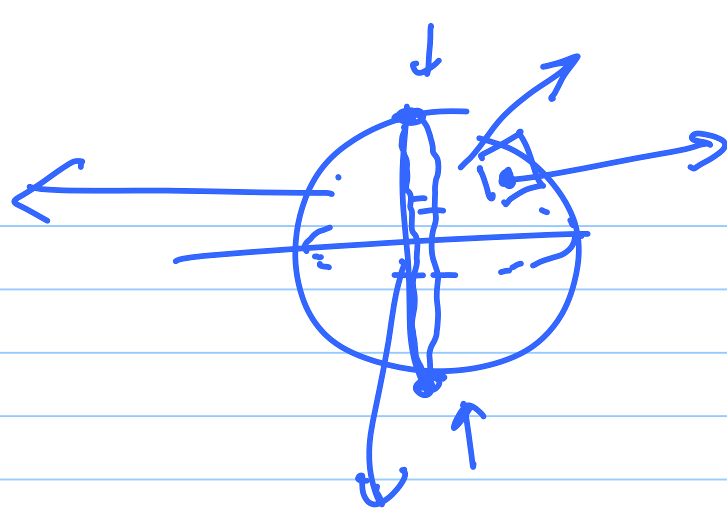
$[\phi] \circ [\psi^{-1}]$ is also diffm C^∞

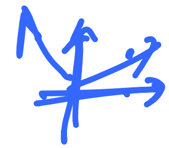


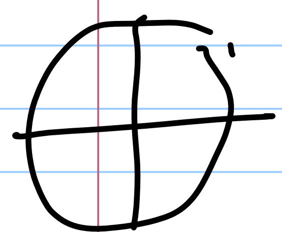
$X =$

Manifold: a set $S \subseteq \mathbb{R}^N$

UNION of overlapping charts = Atlas







left hemisphere:

$\Phi : \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

$\Theta : (0, \pi)$

Chart : $U \xrightarrow{\phi} X$

$\underline{x, y, z} \rightarrow \phi, \theta$

$(x, \cancel{y}, \sqrt{1-x^2-z^2})$

$x^2 + y^2 = 1$ $\leftarrow$ set

$x^2 + y^2 + z^2 = 1$ $\leftarrow$ Implicit Eqn.

$-1 = 0$

$\underline{a} = (x_1, \dots x_n)$

$\underline{b} = (x_{n+1}, \dots x_{n+m})$

$z = \sqrt{1-x^2-y^2}$

$\underline{a} = n \times 1$

$\underline{b} = m \times 1$

$f : \mathbb{R}^{n+m} \longrightarrow \mathbb{R}^m$

<u>Implicit func. theorem:</u>

$\underline{f}(\underline{a}, \underline{b}) = \underline{0}$

$$\underline{f} = \begin{pmatrix} f_1 \\ \vdots \\ f_m \end{pmatrix} \qquad J = \begin{bmatrix} \frac{\partial f_1}{\partial x_{n+1}} & \cdots & \frac{\partial f_m}{\partial x_{n+m}} \\ \vdots & & \\ \frac{\partial f_m}{\partial x_{n+1}} & \cdots & \frac{\partial f_m}{\partial x_{n+m}} \end{bmatrix}$$

$\underline{x_{n+1} \dots x_{n+m}} : \underline{g}(x_1, \dots x_n)$

Tangent space:

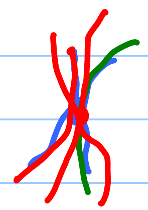


chart $U \subseteq \mathbb{R}^n \underset{\phi^{-1}}{\overset{\phi}{\rightleftarrows}} X \subseteq \mathbb{R}^m$

$$\begin{array}{ccc} \gamma & \xrightarrow{\phi} & \beta \\ = & & = \\ \frac{d\gamma}{ds} & & \frac{d\beta}{ds} \end{array}$$

→ basis induced by chart

|| later we will see others basis for motion

tangent bundle $\begin{pmatrix} p \\ p_t \end{pmatrix}$ ← statespace

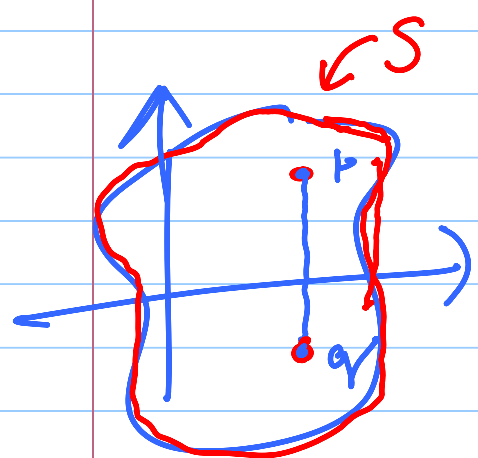
we will see/use this material when we deal with c-space.

Computational Geometry

Basic Notions

2-D or 3-D geometric entities linear, planar

1) Convexity :



S is convex iff

$$\forall \underline{p}, \underline{q} \in S \Rightarrow \alpha \underline{p} + (1-\alpha)\underline{q} \in S$$

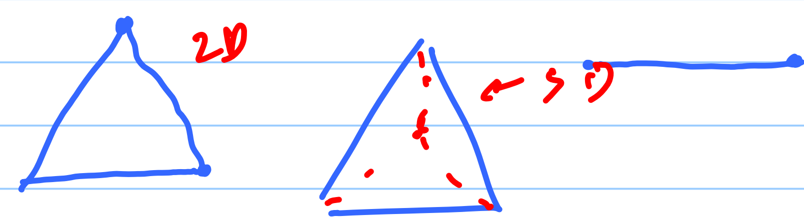
$$0 \leq \alpha \leq 1$$

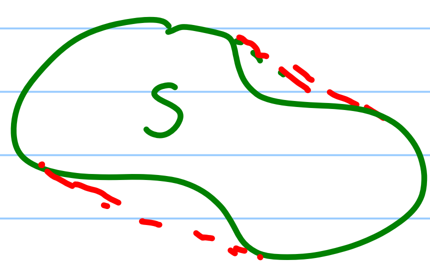
2) Convex comb. of pts:

$x_1, x_2 \ldots x_n$

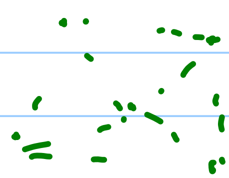
$$\sum_{i=1}^{n} \alpha_i x_i \qquad 0 \le \alpha_i \le 1$$

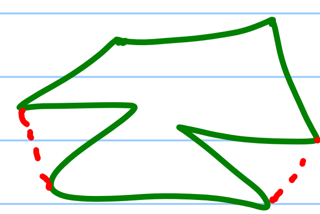
$$\sum \alpha_i = 1$$





What is the smallest convex set that encloses S : Convex Hull

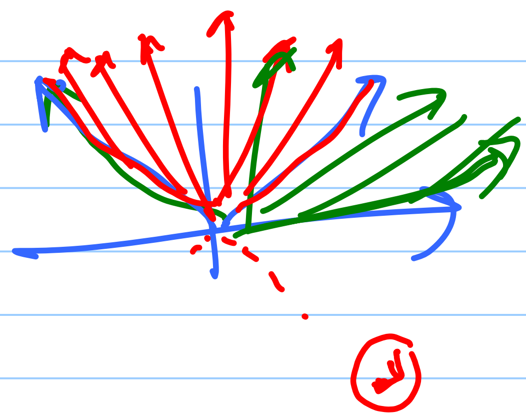




Geom.

Convex Cone: Let C be non-empty set of vectors $\hat{s}_i$

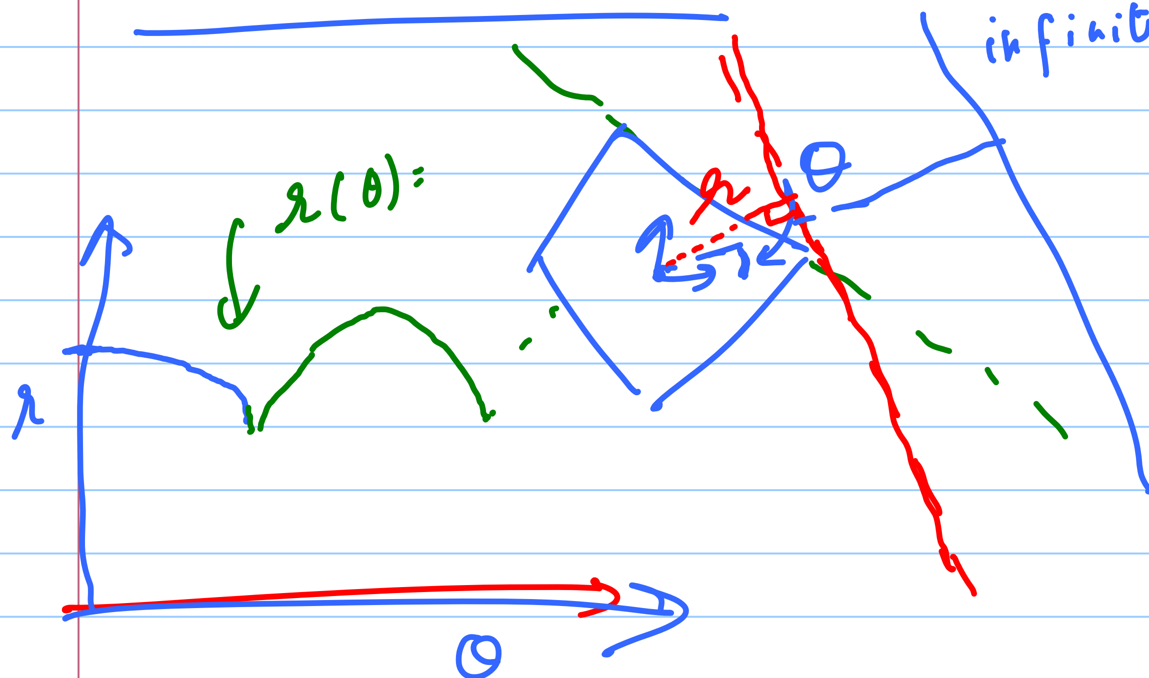
$$C^{<} : \left\{ \hat{s} : \hat{s} = \sum_i \alpha_i \hat{s}_i \quad \alpha_i \geq 0,\ \hat{s}_i \in C \right]$$



$C_1^{<} \cup C_2^{<}$

Caratheodory Th: at least $p+1$ vectors are needed to +vely span the p-dim. space will ~~see~~ use this for grasping

Radius function: Convex Sets

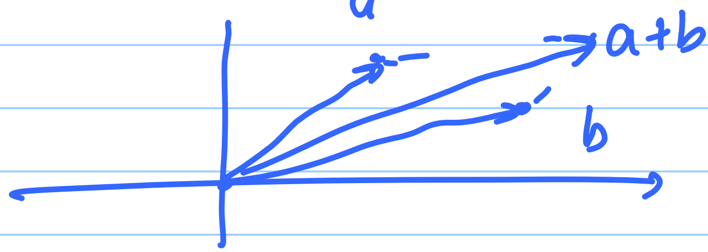


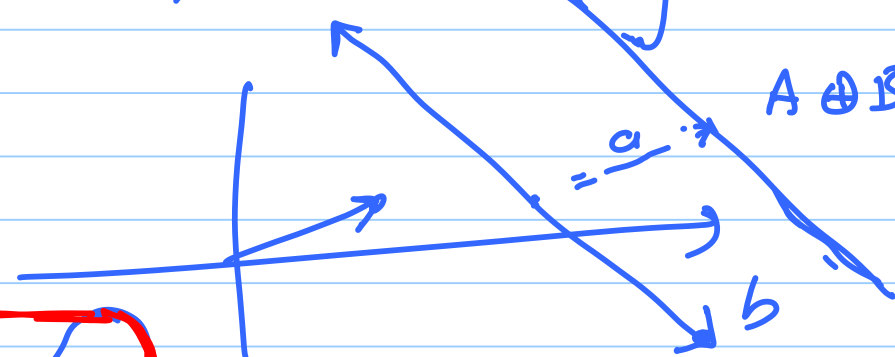
for grasping very useful.

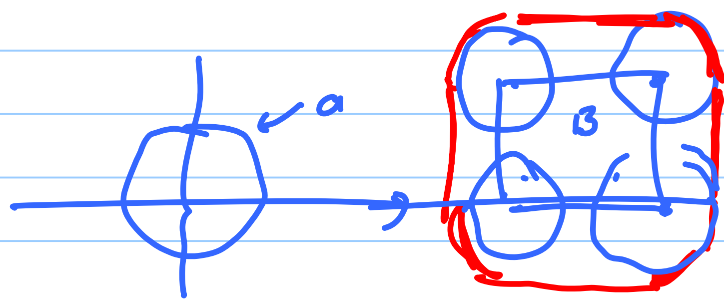
Minkowski sum:

A, B are two affine sets in an affine space

$$A \oplus B = \{ a+b, \ \forall a \in A, \ \forall b \in B \}$$







"growing operation"

A, B are convex compact sets

$\Pi(A) \rightarrow$ Boundary of a set A

$$\Pi(A \oplus B) = \Pi(A) \oplus \Pi(B)$$

for convex polys, eff. alg. ~~to~~ det. minkowski sum
linear in number of vertices

Voronoi Diagrams:

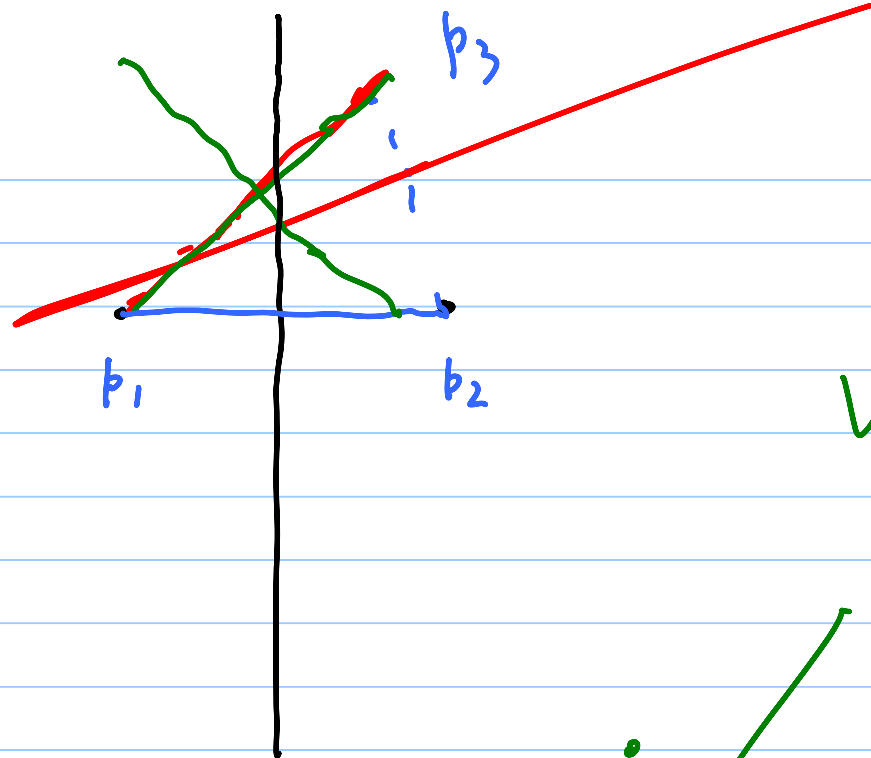
Dual

Delauney

Triangulation:

Shape : expressig it as a union of disjoint cells

triangles



$$V(p_i) = \{x : d(p_i, x) \le d(p_j, x) \ \forall j \neq i\}$$